

1 A curve has implicit equation  $y^2 + 2x \ln y = x^2$ .

Verify that the point (1, 1) lies on the curve, and find the gradient of the curve at this point. [6]

2 A curve has equation  $x^2 + 2y^2 = 4x$ .

(i) By differentiating implicitly, find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ . [3]

(ii) Hence find the exact coordinates of the stationary points of the curve. [You need not determine their nature.] [3]

3 Given that  $y = \ln\left(\sqrt{\frac{2x-1}{2x+1}}\right)$ , show that  $\frac{dy}{dx} = \frac{1}{2x-1} - \frac{1}{2x+1}$ . [4]

4 Fig. 7 shows the curve  $x^3 + y^3 = 3xy$ . The point P is a turning point of the curve.

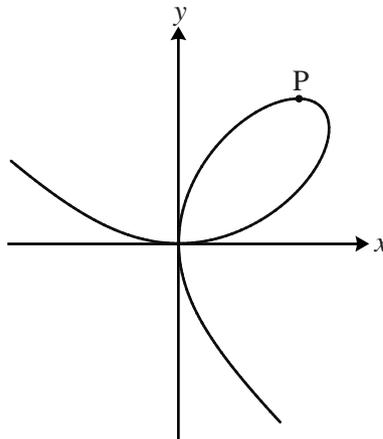


Fig. 7

(i) Show that  $\frac{dy}{dx} = \frac{y-x^2}{y^2-x}$ . [4]

(ii) Hence find the exact  $x$ -coordinate of P. [4]

5 Find the gradient at the point  $(0, \ln 2)$  on the curve with equation  $e^{2y} = 5 - e^{-x}$ . [4]

6 A curve is defined by the equation  $(x + y)^2 = 4x$ . The point  $(1, 1)$  lies on this curve.

By differentiating implicitly, show that  $\frac{dy}{dx} = \frac{2}{x + y} - 1$ .

Hence verify that the curve has a stationary point at  $(1, 1)$ . [4]

7 A curve is defined by the equation  $\sin 2x + \cos y = \sqrt{3}$ .

(i) Verify that the point  $P\left(\frac{1}{6}\pi, \frac{1}{6}\pi\right)$  lies on the curve. [1]

(ii) Find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ .

Hence find the gradient of the curve at the point  $P$ . [5]

8 (i) Given that  $y = \sqrt[3]{1 + 3x^2}$ , use the chain rule to find  $\frac{dy}{dx}$  in terms of  $x$ . [3]

(ii) Given that  $y^3 = 1 + 3x^2$ , use implicit differentiation to find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ . Show that this result is equivalent to the result in part (i). [4]